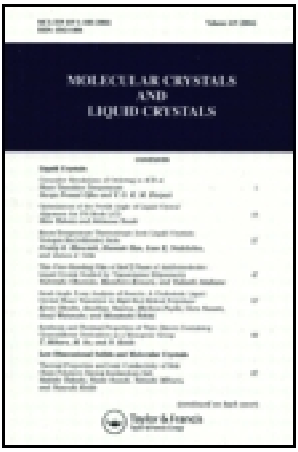

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Optical Defect Modes in Chiral Liquid Crystals at Birefringent Defect Layer

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An analytic approach to the theory of the optical defect modes in chiral liquid crystals (CLC) for the case of a birefringent defect layer is developed. The analytic study is facilitated by the choice of the problem parameters. Namely, a birefringent layer (with the averaged dielectric susceptibility equal to the average dielectric susceptibility of CLC) sandwiched between two CLC layers inserted in an isotropic medium with the dielectric constant equal to the average dielectric constant of CLC is studied. The chosen model allows one to get rid off the polarization mixing at the external surfaces of the defect structure (DMS) and to reduce the corresponding equations to the equations for light of diffracting in CLC polarization only. The dispersion equation determining connection of the defect mode frequency with the layer birefringence and other parameters of the defect structure is obtained. Analytic expressions for the transmission and reflection coefficients of the defect mode structure (CLC- birefringent defect layer-CLC) are presented and analyzed. It is shown that generally speaking the layer birefringence reduces the DM lifetime in comparison with the DMS with an isotropic defect layer and only at discrete values of DMS parameters it achieves the value of the corresponding DMS with an isotropic defect layer. Correspondingly, generally speaking, the effect of anomalously strong light absorption at the defect mode frequency and the lowering of the lasing threshold are not so pronounced as in the case of the DMS with an isotropic defect layer. The case of DMS with low defect layer birefringence is studied in details. The options of effectively influence at the DM parameters by changing the defect layer birefringence are discussed.

Keywords Birefringent defect layer; localized modes in CLC; low threshold DFB lasing

Introduction

Recently there was an intense activity in the field of mirrorless distributed feedback (DFB) lasing in structures consisting from many layers of chiral liquid crystals (CLC) mainly due to the possibilities to reach a low lasing threshold for the DFB lasing [1–8]. The most part of the related theory is based at the numerical calculations [9] which results are not always interpreted in the frame work of a clear physical picture [A]. Several recent papers [10–13] showed that an analytic theoretical approach to the problem (some times being limited by the introduced approximations) allows to create a clear physical picture of the linear optics and lasing in the mentioned structures. In particular, the role of localized

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optical modes (edge and defect modes) in the structures under consideration was clearly demonstrated. The most promising results in DFB lasing relates to defect modes (DM) [12,13]. The defect modes existing at the structure defect as a localized electromagnetic eigen state with its frequency in the forbidden band gap were investigated initially in the three-dimensionally periodic dielectric structures [14]. The corresponding defect modes in chiral liquid crystals, and more general in spiral media, are very similar to the defect modes in one-dimensional scalar periodic structures. They reveal abnormal reflection and transmission inside the forbidden band gap [1,2] and allow DFB lasing at a low lasing threshold [3]. The qualitative difference with the case of scalar periodic media consists in the polarization properties. The defect mode in chiral liquid crystals is associated with a circular polarization of the electromagnetic field eigen state of the chirality sense coinciding with the one of the chiral liquid crystal helix. There are two main types of defects in chiral liquid crystals studied up to now. One of them is a plane layer of some substance differing from CLC dividing in two parts a perfect cholesteric structure and being perpendicular to the helical axes of the cholesteric structure [1]. Other one is a jump of the cholesteric helix phase at some plane perpendicular to the helical axes (without insertion any substance at the location of this plane) [2]. Recently, a lot of new types of defect layer were studied [15–22], for example, the CLC layer with the pitch differing from the pitch of two layers sandwiching the defect layer [8]. It is evident that there are many versions of the dielectric properties of the defect layer. However, the consideration below will be limited by the first mentioned above types of defect, namely, a birefringent layer inserted in a chiral liquid crystal. Main attention will be paid to a birefringent defect layer with low birefringence. The reason for that is connected as with the experimental researches of the DFB lasing in CLC where a defect layer is birefringent [22] so with a general idea that the unusual properties of DM manifest themselves most clearly just at the middle of defect structure, i.e. at defect layer where intensity of the DM field reaches its maximum. we shell assume also at th beginning that there is no absorption in the CLC and birefringent defect. The analytic approach in studying of a DMS with a birefringent defect layer is very similar to the previously performed DM studies for isotropic defect layer [12,13], so we shell present below the final results of the present investigation sending the readers for the investigation details to references [12,13].

In the present paper an analytical solution of the defect mode associated with an insertion of a birefringent defect layer in the perfect cholesteric structure is presented for light propagating along the helical axes and some limiting cases simplifying the problem are considered.

Defect Mode at Birefringent Defect Layer

To consider the defect mode associated with an insertion of a birefringent layer in the perfect cholesteric structure we have to solve Maxwell equations and a boundary problem for electromagnetic wave propagating along the cholesteric helix for the layered structure depicted at Fig. 1. This investigation was performed in [12, 13] under assumption that the defect layer at Fig. 1 is isotropic. So, it is possible to exploit the results of [12,13] for our case of a birefringent defect layer and nonabsorbing and amplifying CLC layers (conserving the main notations of the papers [12,13] here) introducing only some physically clear changes in the formulas obtained in [12,13]. The assumption of [12,13] that the polarization conversion is absent and only light of diffracting circular polarization has to be taken into the consideration (due the assumption that the average dielectric constant of CLC

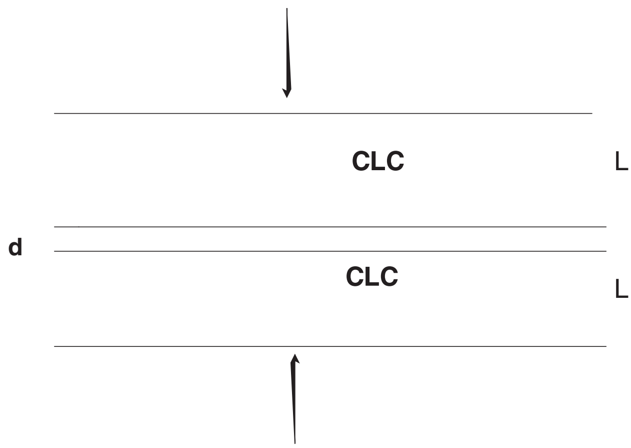


Figure 1. Schematic of the CLC defect mode structure with birefringent defect layer of thickness d.

ε_0 is coinciding with the dielectric constant of defect layer and external media) is not valid here. Really, due to the birefringence of the defect layer the light polarization is changing in the course of it propagation in the defect layer from one its surface to the second one so, generally speaking, the light polarization after crossing the defect layer differs from the light polarization at the first defect layer surface. It is why the polarization component differing from the diffracting polarization, generally speaking, occurs to be present and a leakage of the correspondingly polarized light from the DMS takes place. The evident consequence of this leakage is, in general, reduction of the DM life-time in the case of a birefringent defect layer.

As is known [9] a lot of information on DM is available from spectral properties of DMS transmission T(d, L) and reflection R(d, L) coefficients, where d and L are the defect layer and CLC layers thicknesses.

There is an option to obtain formulas determining the optical properties of the structure depicted at Fig. 1 using the expressions for the amplitude transmission T(L) and reflection R(L) coefficient for a single cholesteric layer in the presence of dielectric interfaces (see [23,24]). If one neglects the multiple scattering of nondifracting polarization light the transmission $\left|T(d, L)^2\right|$ and reflection $|R(d, L)|^2$ intensity coefficients (of light of diffracting circular polarization) for the whole structure may be presented in the following form:

$$|T(d, L)|^2 = |[T_e T_d M(k, d, \Delta n)(\boldsymbol{\sigma}_e \boldsymbol{\sigma}^*_{ed})]/[1 - M^2(k, d, \Delta n)(\boldsymbol{\sigma}_r \boldsymbol{\sigma}^*_{ed})^2 (R_d R_u]|^2, \quad (1)$$

$$|R(d, L)|^2 = |\{R_e + R_d\ T_e T_u M^2(k, d, \Delta n)|\boldsymbol{\sigma}_e \boldsymbol{\sigma}^*_{ed}|^2/[1 - M^2(k, d, \Delta n)|\boldsymbol{\sigma}_r \boldsymbol{\sigma}^*_{ed}|)^2 \times R_d R_u]\}|^2, \quad (2)$$

where $R_e(T_e)$, $R_u(T_u)$ and $R_d(T_d)$ are the amplitude reflection (transmission) coefficients of the CLC layer (see Fig. 1) for the light of diffracting polarization incident at the outer (top) layer surface, for the light incidence at the inner top CLC layer surface from the inserted defect layer and for the light incidence at the inner bottom CLC layer surface from the inserted defect layer, respectively, $\boldsymbol{\sigma}_e$, $\boldsymbol{\sigma}_r$ and $\boldsymbol{\sigma}_{ed}$ are the polarization vectors of light exiting the CLC layer inner surface, reflected at the inner bottom CLC layer surface at the incidence from the inserted defect layer and of light whose some polarization vector $\boldsymbol{\sigma}_{ed}$ transforms to the polarization vector $\boldsymbol{\sigma}_e$ at crossing the birefringent defect layer of thickness d, respectively, Δn is the difference of two refractive indexes in the birefringent defect layer and M(k,d, Δn) is the phase factor related to light single propagation in a birefringent defect layer. It is assumed in the deriving of Eqs.(1,2) that the external beam is incident at the structure (Fig. 1) from the above only. In the presence of dielectric interfaces the light polarization at the inner surfaces of CLC layers in DMS at reflection and transmission of light through a CLC layer ceased to be purely circular for the light field inside CLC layers presented as a superposition of only two eigen diffracting modes of CLC. The corresponding polarization vector may be found (see [23,24]) as well as the polarization vector $\boldsymbol{\sigma}_{\mathbf{ed}}$ may be easily calculated if the d, and Δn are known. The same should be said about the polarization of light exciting in a CLC layer only the diffracting eigen modes in its incidence at the external CLC layer surface in DMS. The corresponding polarization at the presence of dielectric interfaces is named here a diffracting polarization. Polarization orthogonal to the diffracting polarization is called here a nondiffracting polarization. Light of nondiffracting polarization being incident at a DMS excites in the CLC layers of DMS only nondifracting eigen CLC modes. The polarization vectors $\boldsymbol{\sigma}_{\mathbf{e}}$, $\boldsymbol{\sigma}_{\mathbf{r}}$ and $\boldsymbol{\sigma}_{\mathbf{ed}}$ may be presented in the form:

$$\boldsymbol{\sigma}_i = (\cos\alpha_{\rm i}\mathbf{e}_{\rm x} + \mathbf{e}_{\rm i}^{{\rm i}\beta}\ \sin\alpha_{\rm i}\mathbf{e}_{\rm y}), \tag{3}$$

where $\mathbf{e}_{\rm x}$ and $\mathbf{e}_{\rm y}$ are the unit vectors along the x and y axis and $\alpha_{\rm i}$, $\beta_{\rm i}$ are the parameters determining the polarization. For example, $\alpha_{\rm i} = \pi/4$ and $\beta_{\rm i} = \pi/2(-\pi/2)$ corresponds to right (left) circular polarization.

In the general case transmitted and reflected beams do not correspond to the diffracting polarization for a birefringent defect layer so there is reflection and transmission of the light of nondiffracting polarization even for incident light of diffracting polarization. Neglecting the multiple scattering of nondiffracting polarization light one gets the corresponding expression for reflection $\mathrm{R(d, L)}^-$ and transmission $\mathrm{T(d, L)}^-$ coefficients of light of nondiffracting polarization (for incident light of diffracting polarization):

$$\begin{aligned}|\mathrm{T(d, L)}^-|^2 = \big|[\mathrm{T_e T_d^-}]\big\{\mathrm{M(k, d, \Delta n)}\big(\boldsymbol{\sigma}_{\mathbf{e}}\boldsymbol{\sigma}_{\mathbf{ed}}^{\perp}*\big) + (\boldsymbol{\sigma}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{ed}}^{*})(\boldsymbol{\sigma}_{\mathbf{e}}\boldsymbol{\sigma}_{\mathbf{ed}}^{*})\\ \times\big(\boldsymbol{\sigma}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{ed}}^{\perp}*\big)\mathrm{M^2(k, d, \Delta n)}/[1-\mathrm{M^2}](\mathrm{k, d, \Delta n})(\boldsymbol{\sigma}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{ed}}^{*})^2(\mathrm{R_d R_u})\big\}\big|^2,\end{aligned} \tag{4}$$

$$\begin{aligned}|\mathrm{R(d, L)}^-|^2 = \big|\big\{\mathrm{R}_e^- + \mathrm{R_d T_e T_u^- M^2(k, d, \Delta n)}(\boldsymbol{\sigma}_{\mathbf{e}}\boldsymbol{\sigma}_{\mathbf{ed}}^{*})\big(\boldsymbol{\sigma}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{ed}}^{\perp}*\big)/[1-\mathrm{M^2(k, d, \Delta n)}\\ \times(\boldsymbol{\sigma}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{ed}}^{*})^2\mathrm{R_d R_u}]\big\}\big|^2,\end{aligned} \tag{5}$$

where $\mathrm{R_e^-}$ is the reflection coefficients of the CLC layer for the light of nondiffracting polarization taking into account dielectric interfaces at incidence of light of diffracting polarization and $\mathrm{T^-}$ is transmission coefficients of the CLC layer for the light of nondiffracting polarization taking into account dielectric interfaces at incidence of light of nondiffracting polarization and $\mathbf{e}_{\mathbf{ed}}^{\perp}$ is the polarization vector orthogonal to $\boldsymbol{\sigma}_{\mathbf{ed}}$. Note, that the amplitude transmission coefficients $\mathrm{T}_{\rm d}^-$ and $\mathrm{T}_{\rm u}^-$ are approximately equal to exp[ikLn/$\mathrm{n_0}$], where $\mathrm{n_-}$ is the refractive index of the light of nondiffracting polarization in CLC layer.

Calculations of the reflection and transmission coefficients according (1–2, 4–5) is performable analytically in the general case, however, it is rather cumbersome. It is why below will be studied in details the case of low birefringence and presented expressions for $\left|\mathrm{T(d, L)}^2\right|$ and $|\mathrm{R(d, L)}|^2$ taking into account polarization transformation in the defect layer only and neglecting transformations of polarizations at the interfaces and their small deviations from circular one allowing simple analytical calculations.

Under the mentioned above simplification and the assumption that the refractive indexes of the DMS external media coincides with the average CLC refractive index and the refractive indexes of defect layer may be given by the formulas

$$n_{max} = n_0 + \Delta n/2, \quad n_{min} = n_0 - \Delta n/2, \tag{6}$$

where n_0 coincides with the average CLC refractive index and Δn is small the phase factor M(k, d, Δn) is presented by the following expression:

$$M(k, d, \Delta n) = \exp[ikd] \cos(\Delta\varphi/2), \tag{7}$$

where the phase difference of two beam component with different eigen polarization at the defect layer thickness is $\Delta\varphi = \Delta nkd/n_0$, $k = \omega n_0/c = \omega\varepsilon_0{}^{1/2}/c$, $\varepsilon_0 = (\varepsilon_\parallel + \varepsilon_\perp)/2$, and $\varepsilon_\parallel$, $\varepsilon_\perp$ are the local principal values of the LC dielectric tensor [23–27].

Finally, the explicit expressions for reflection and transmission coefficients of light with a circular diffracting polarization for the incident beam with a circular diffracting polarization in the case of low birefringence one gets inserting (7) into (1–2):

$$|T(d, L)|^2 = |[T_eT_d \exp[ikd] \cos(\Delta\varphi/2)]/[1 - \exp[i2kd] \cos^2(\Delta\varphi/2)R_dR_u]|^2, \tag{8}$$

$$|R(d, L)|^2 = |\{R_e + R_dT_eT_u \exp[i2kd] \cos^2(\Delta\varphi/2)/[1 - \exp[i2kd] \cos^2 \times (\Delta\varphi/2)R_dR_u]\}|^2 \tag{9}$$

We, for the sake of completeness, present here also the expressions for transmission T(L) and reflection R(L) coefficient for a single cholesteric layer of thickness L for light of diffracting circular polarization for the case of dielectric constant of the external medium coinciding with the average CLC dielectric constant [23,24]:

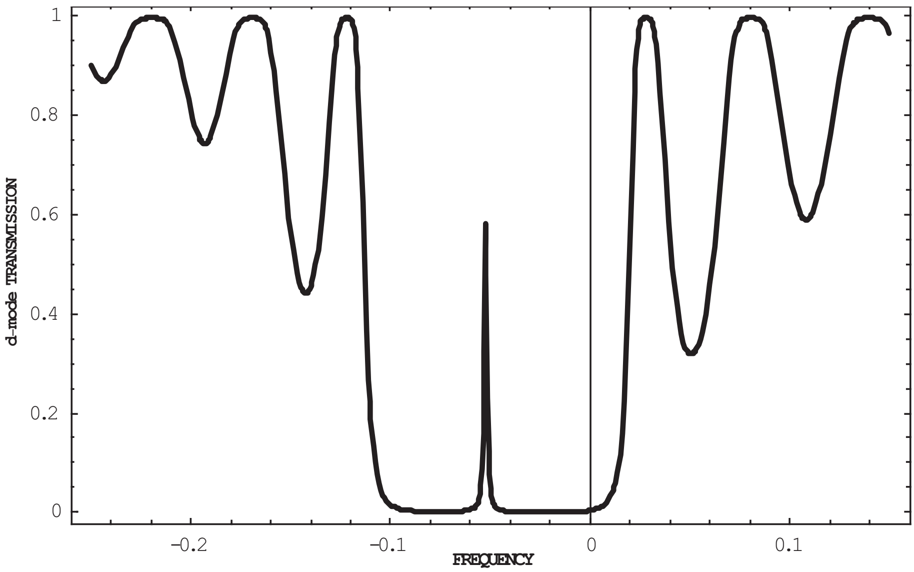
$$\begin{aligned} R(L) &= i\delta \sin qL/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\} \\ T(L) &= \exp[i\tau L/2](q\tau/\kappa^2)/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}, \end{aligned} \tag{10}$$

where $\tau = 4\pi/p$, p is the cholesteric pitch, and $\delta = (\varepsilon_\parallel - \varepsilon_\perp)/(\varepsilon_\parallel + \varepsilon_\perp)$ is the dielectric anisotropy,

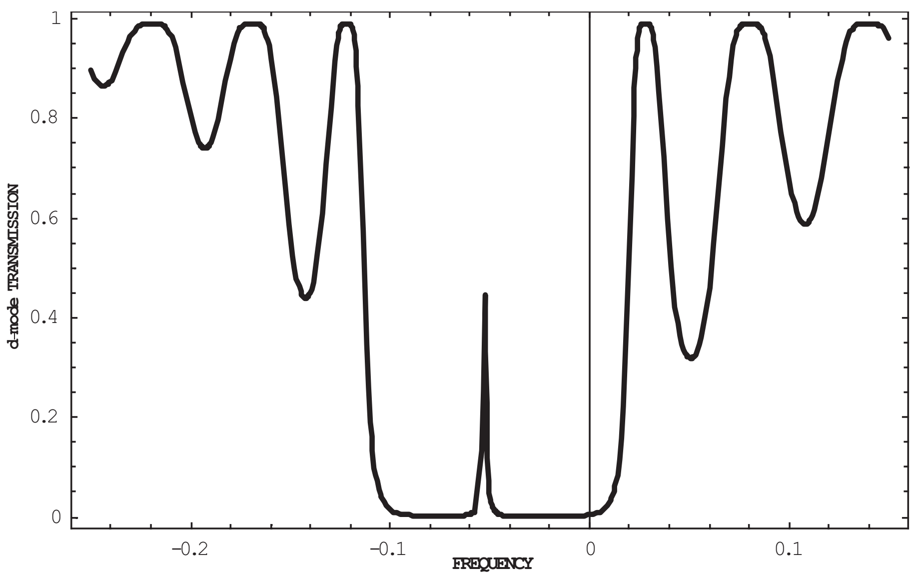
$$q = \kappa\{1 + (\tau/2\kappa)^2 - [(\tau/\kappa)^2 + \delta^2]^{1/2}\}^{1/2}.$$

The Eqs. (8,9) show that if $\Delta\varphi/2\pi$ is an integer number the Eqs. (8,9) are identical to the corresponding equations for DMS with an isotropic defect layer [12,13] and there is no conversion of diffracting polarization into nondifracting one, however if $\Delta\varphi/2\pi$ is not an integer number a conversion of diffracting polarization into nondifracting one happens and, consequently, a leakage of the light from the DMS takes place and,I n particular, the DM life-time is less than for the case of the corresponding DMS with an isotropic defect layer. This dependence of the DMS properties on the phase shift between eigen waves at their crossing the defect layer opens the ways to control the DM properties. The simplest one is related to variations of the defect layer thickness.

The polarization conversion results in addition of the nondiffracting components to the transmitted and reflected beams. For the case of low birefringence, what corresponds to the condition $\Delta n/n_0 < \delta$, the amplitude transmission and reflection coefficient for the light of nondiffracting polarization (for the incident light of diffracting polarization) are given by



(a)



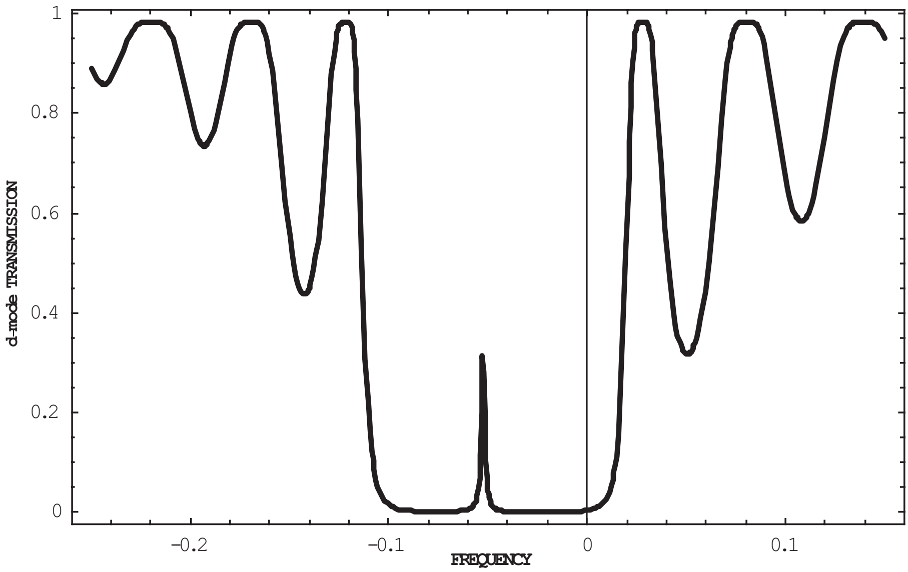
(b)

Figure 2. Calculated diffracting polarization intensity transmission coefficient $|T(d, L)|^2$ for a low birefringent defect layer versus the frequency (Here and at all other figures $\nu = \delta[2(\omega - \omega_B)/(\delta\omega_B) - 1)]$) for a diffracting incident polarization at the birefringent phase shift at the defect layer thickness $\Delta e\mathcal{T} = \pi/20$ (a), $\pi/16$ (b), $\pi/12$ (c), $\pi/8$ (d), $\pi/6$ (e), $\pi/4$ (f), $\pi/2$ (g), and $\Delta e\mathcal{T} = 0$ (h, corresponds to the isotropic defect layer) for a nonabsorbing CLC at d/p = 0.25; $L\tau = 2\pi N$, where here and at all other figures $\delta = 0.05$ and N = 33 is the director half-turn number at the CLC layer thickness L. *(Continued)*

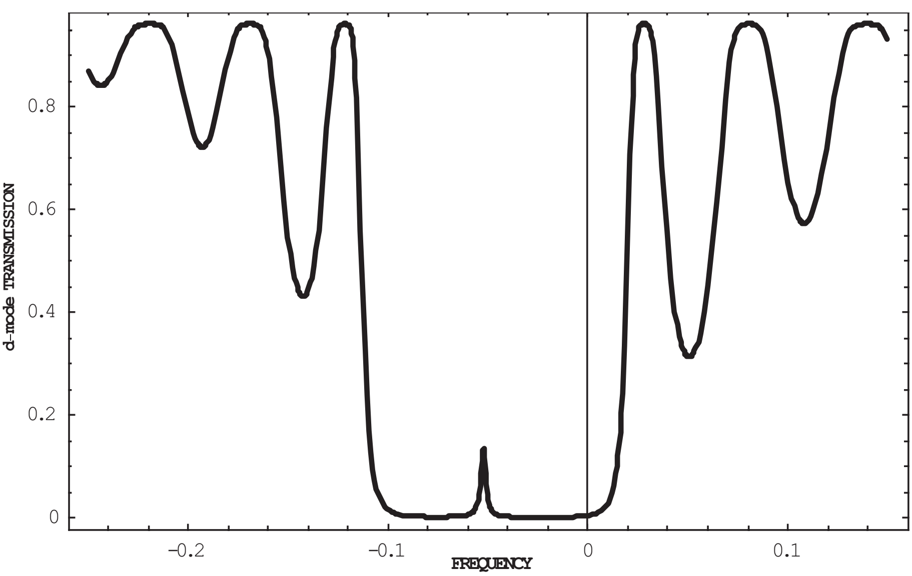
the expressions:

$$T(d, L)^{-} = [T_e \exp[ikLn_{-}/n_0]\exp[ikd]\sin(\Delta \mathscr{er}/2)]/[1 - \exp[i2kd]\cos^2 \times (\Delta \mathscr{er}/2)R_dR_u], \quad (11)$$

$$R(d, L)^{-} = 1/2R_u T_e \exp[ikLn_{-}/n_0]\exp[i2kd]\sin(\Delta \mathscr{er})/[1 - \exp[i2kd]\cos^2 \times (\Delta nkd/2n_0)R_dR_u], \quad (12)$$



(c)

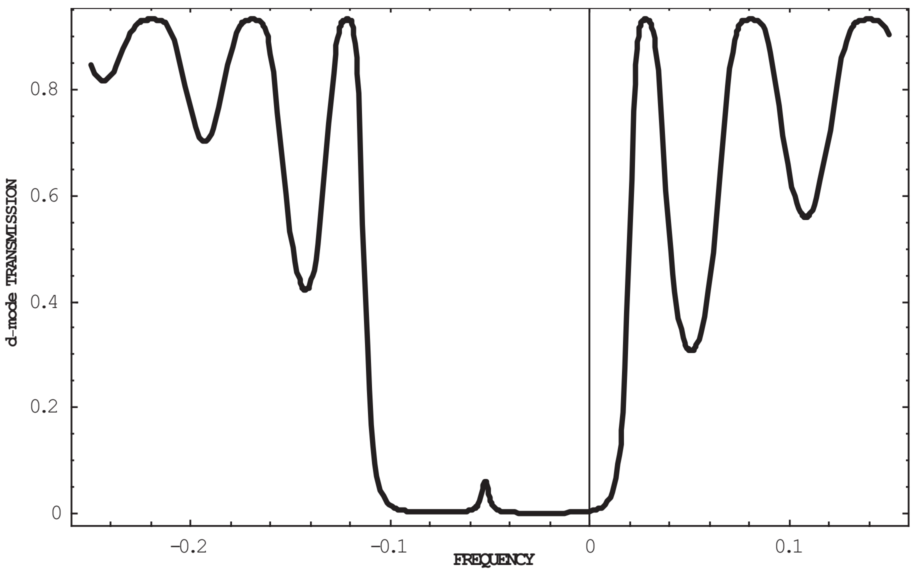


(d)

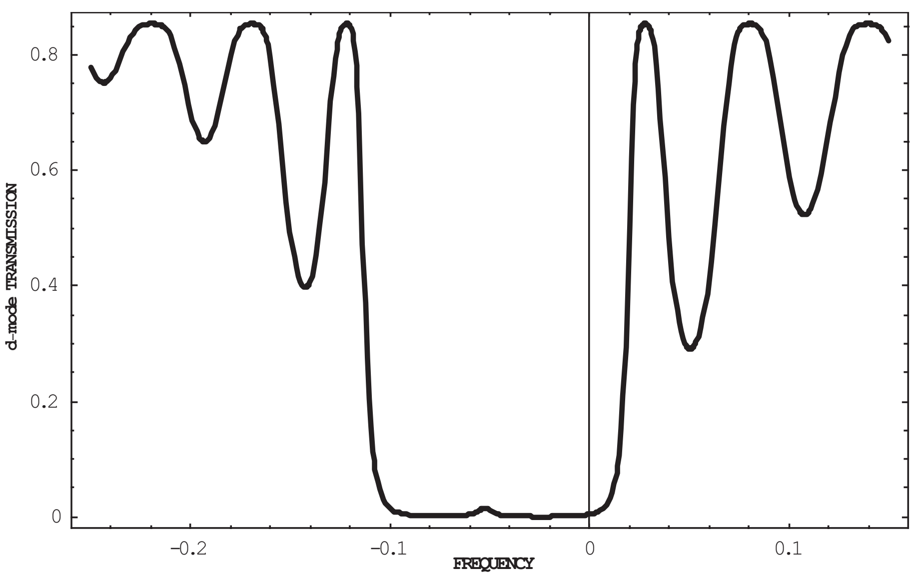
Figure 2. *(Continued)*

where n- is the refractive index of the light of nondiffracting polarization in CLC layer.

The calculations results for transmission $|T(d, L)|^2$ coefficients of light of diffracting polarization for the case of low birefringence are presented at Fig. 2 for various values of the birefringent phase factor $\Delta e\gamma$ related to the light single propagation in a birefringent defect layer. Fig. 2 show that at low values of phase shift between eigen waves at their crossing the defect layer ($\Delta e\gamma < \pi/2$) the shape of transmission curve is very similar to those for

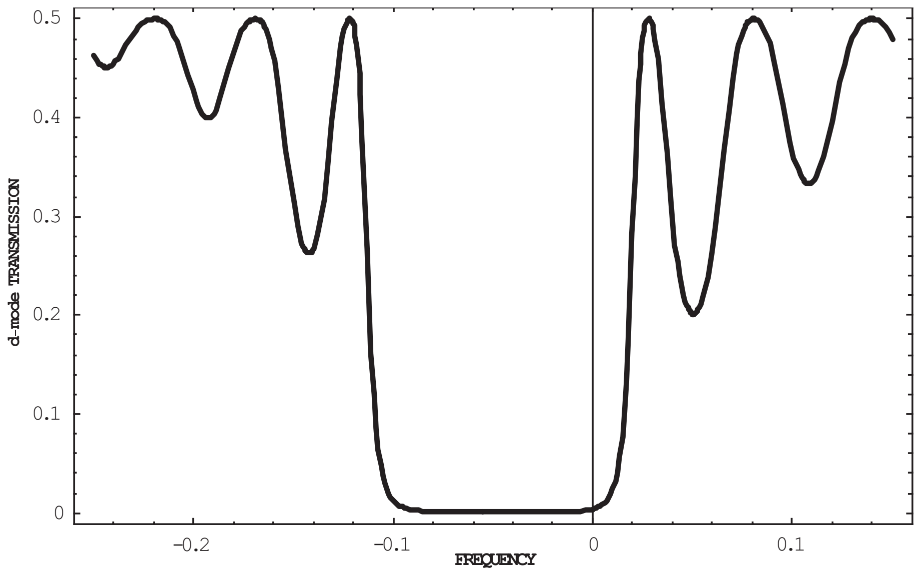


(e)

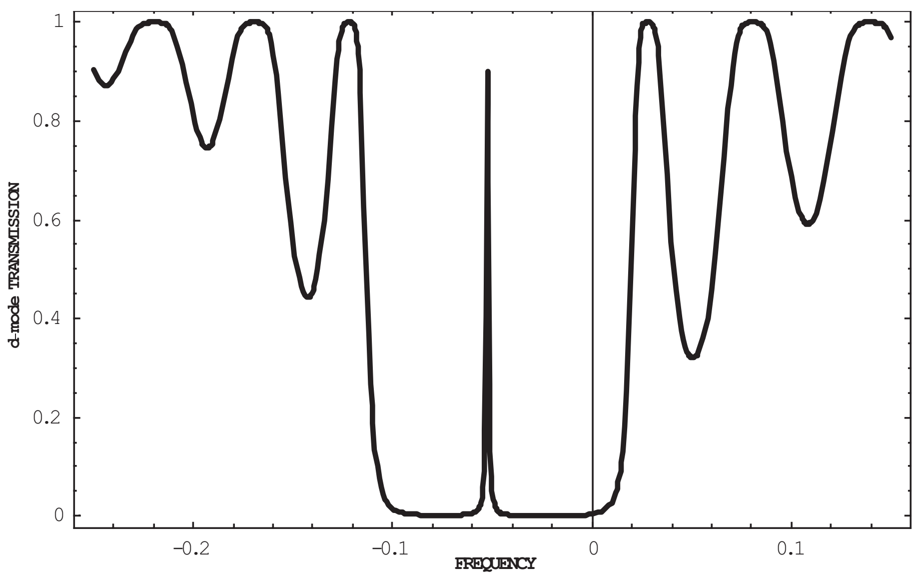


(f)

Figure 2. *(Continued)*



(g)

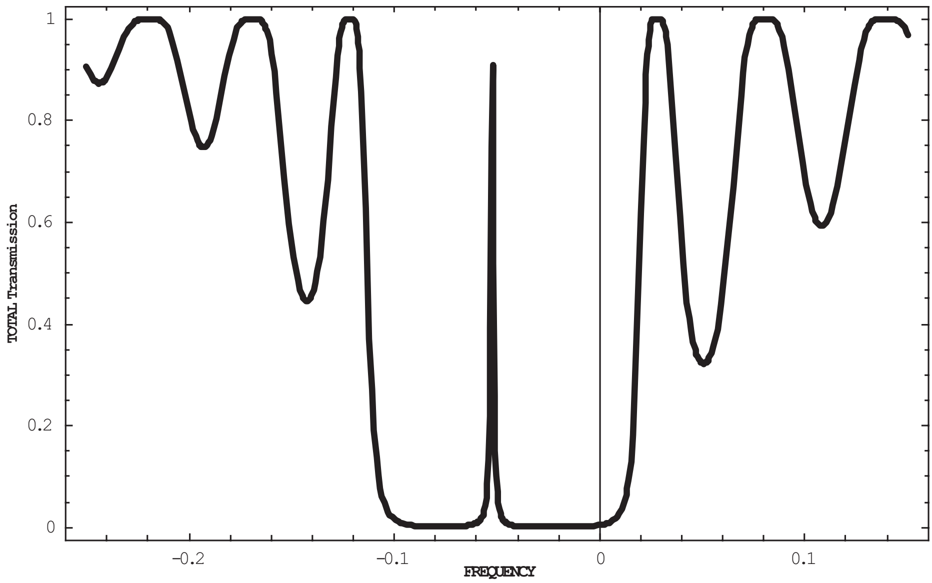


(h)

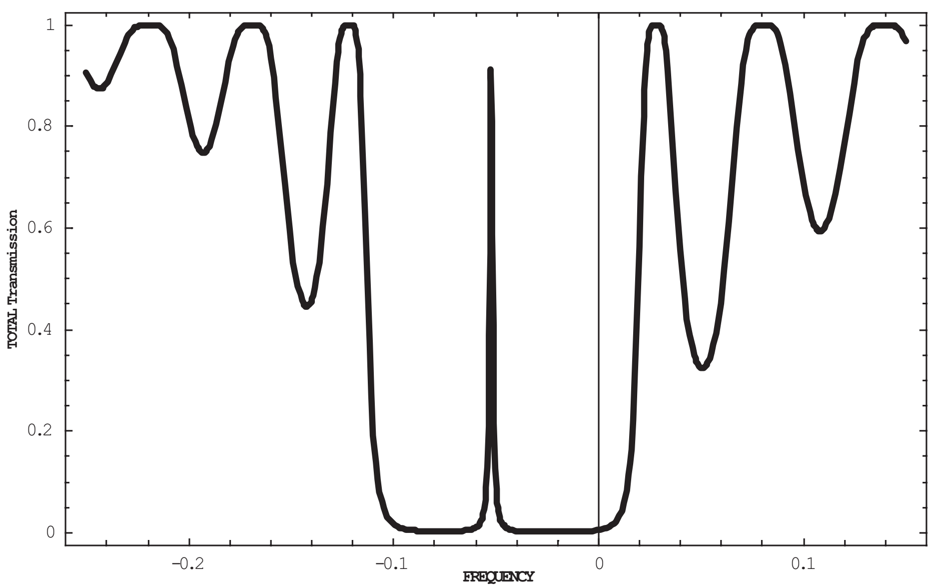
Figure 2. (Continued)

DMS with an isotropic defect layer (for $\Delta e\mathcal{T}$ equal to an integer number or zero it coincides simple with the shape of the corresponding curve for the case of an isotropic defect layer). However, at approaching $\Delta e\mathcal{T}$ to $\pi/2$ (see Figs. 2(e)–(g)) typical for an isotropic defect layer increase of transmission at the defect mode frequency gradually disappears and at $\Delta e\mathcal{T} = \pi/2$ (Fig. 2(g)) does not appear at all. This may be considered, in particular, as a hint that DM life-time is decreasing with increasing of the shift between eigen waves at their crossing the defect layer and at some value of the shift the DM does not exist at all.

If one takes into account the partial conversion of a nondiffracting incident polarization into diffracting one the picture of transmission spectra does not change radically. Really,



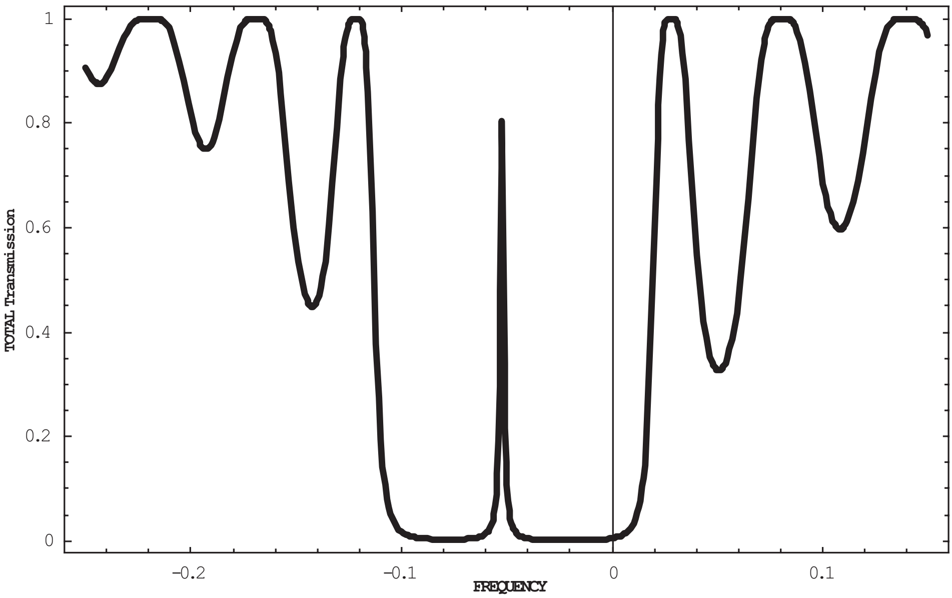
(a)



(b)

Figure 3. Calculated total intensity transmission coefficient for a low birefringent defect layer versus the frequency for diffracting incident polarization at the birefringent phase shift at the defect layer thickness $\Delta e\tau = \pi/20$ (a), $\pi/16$ (b), $\pi/12$ (c), $\pi/8$ (d), $\pi/6$ (e), $\pi/4$ (f), $\pi/2$ (g), for a nonabsorbing CLC at d/p = 0.25. *(Continued)*

the presented at Fig. 3 transmission spectra for total light intensity crossing DMS (for sum of intensities for the both polarizations) calculated with help of Eqs. (11, 12) show a general decrease of transmission at the defect mode frequency ω_d for increase of $\Delta\varphi$ however it is much more slow than for diffracting polarization and only at $\Delta\varphi$ close to $\pi/2$ a practical



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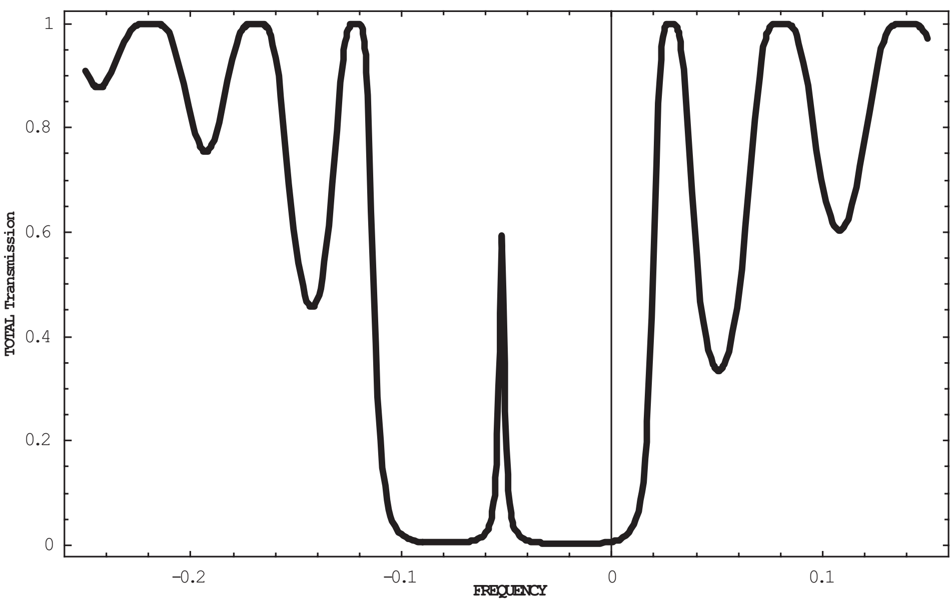
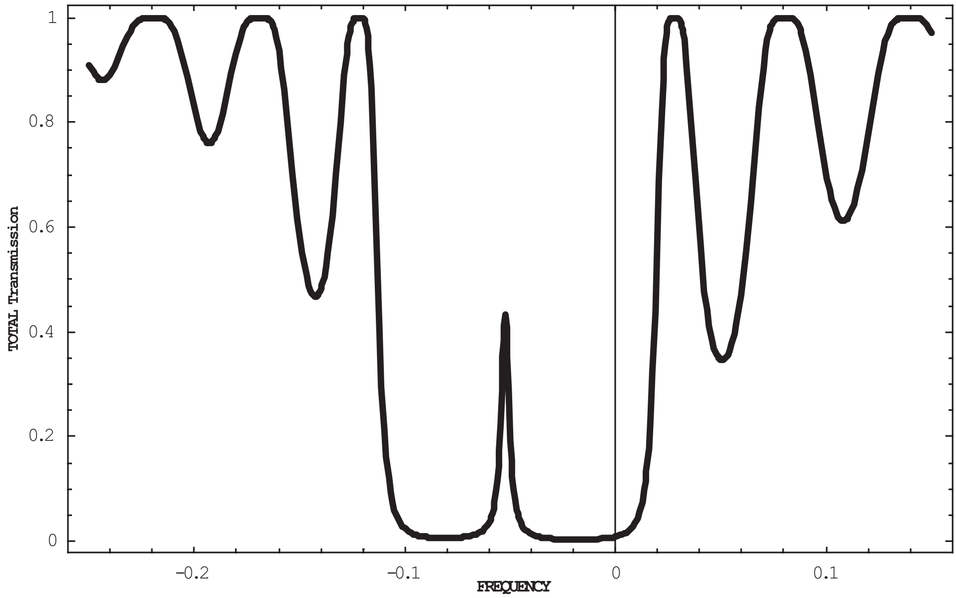


(d)

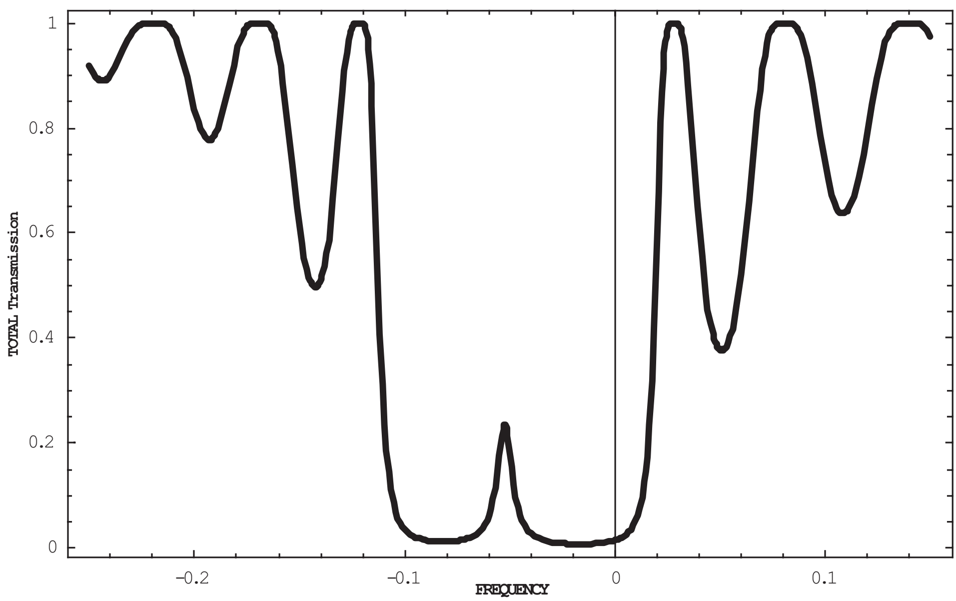
Figure 3. *(Continued)*

absence of the transmission occurs (what demonstrate the conversion of polarization at the birefringent layer).

It iswell known [9] that the position of the edge mode frequency in the stop band is determined by the frequency of the transmission (reflection) coefficient maximum (minimum) so the performed calculation of the transmission spectra (Figs. 2, 3) determine a

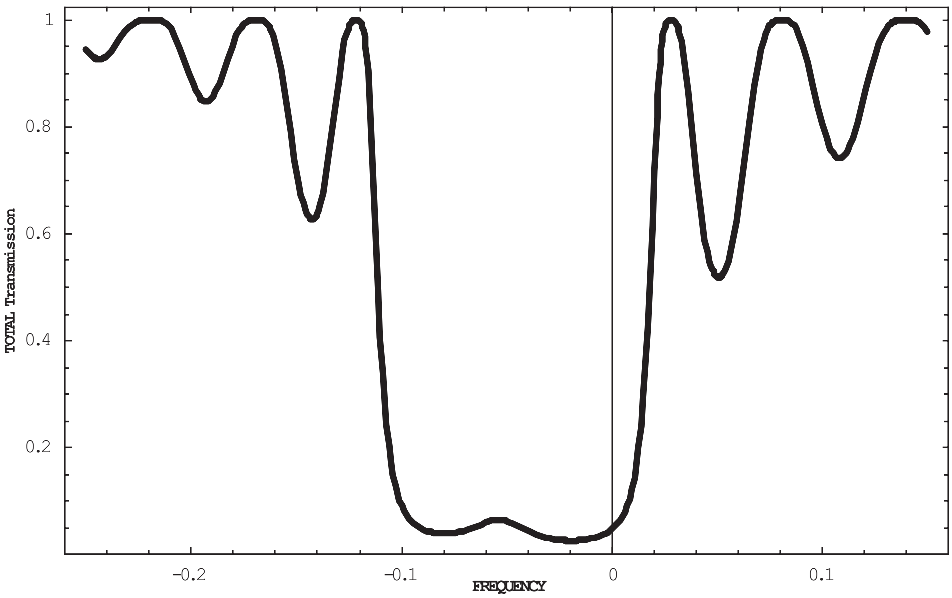


(e)



(f)

Figure 3. *(Continued)*



(g)

Figure 3. (Continued)

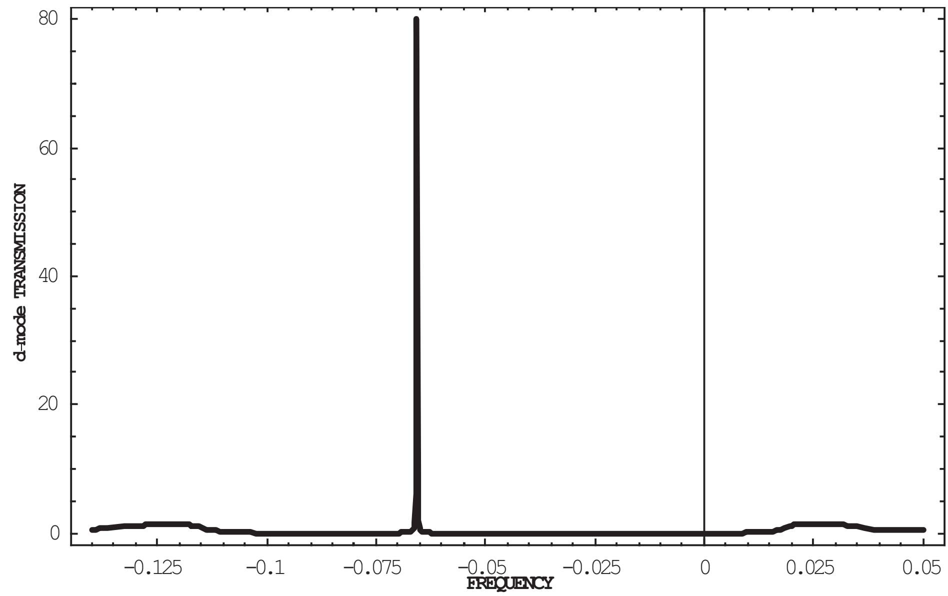
real component of the DM frequency. However because DM is a quasistationary mode an imaginary component of the DM frequency is not zero [12,13]. A direct way to find the imaginary component of the DM frequency is a solving of the dispersion equation. The dispersion equation in the case of a birefringent defect layer may be found similarly to the case of isotropic defect layer [12,13] and if the multiple scattering of light of nondiffracting polarization being neglected it may be presented by the following relationship:

$$\{M^2(k, d, \Delta n)\sin^2 qL - \exp(-i\tau L)[(\tau q/\kappa^2)\cos qL + i((\tau/2\kappa)^2 + (q/\kappa)^2 - 1)\sin qL]^2/\delta^2]\} = 0. \quad (13)$$

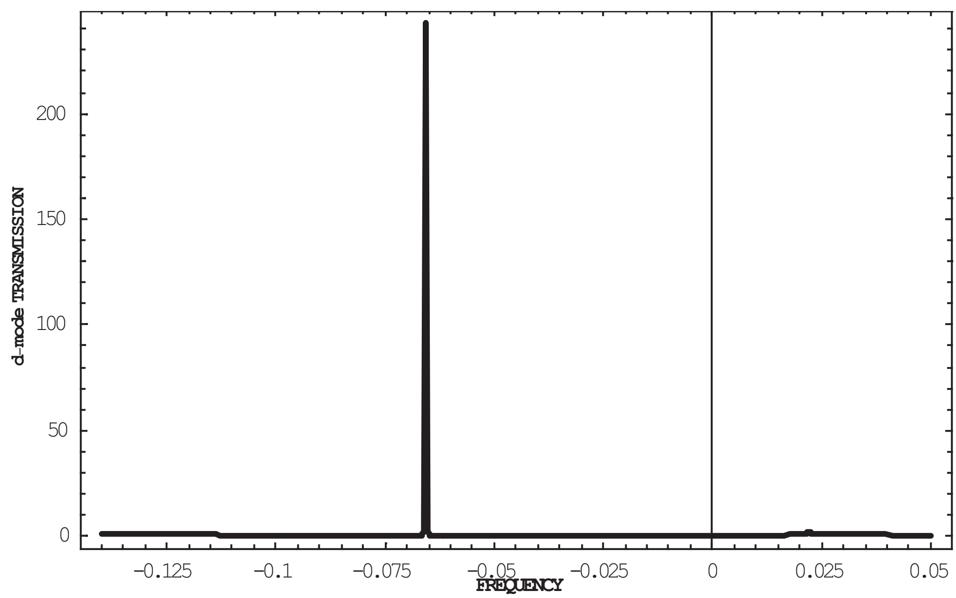
In the general case the solution of equation has to be found numerically and a detailed discussion of this may be found in [12,13]. Some simplification of (13) occurs for the case of low birefringence, when phase factor in (13) is given by (7).

Amplifying and Absorbing CLC Layers

As the experiment [3] and the theory [12,13] show unusual optical properties of DMS at the defect mode frequency ω_D (abnormally strong absorption for absorbing CLC and abnormally strong amplification for amplifying CLC [12,13,23,28]) may be effectively used for enhancement of the DFB lasing. It is quite naturally to study how the birefringent defect layer does influence abnormally strong amplification and abnormally strong absorption effects. For studying this we assume, as was done in [12,13], that the average dielectric constant of CLC has an imaginary addition, i.e. $\varepsilon = \varepsilon_0(1 + i\gamma)$, where positive γ corresponds to absorbing and negative γ to amplifying media (Note, that at real situations $|\gamma| \ll 1$). As was mentioned above the value of γ may be found from solution of the dispersion equation



(a)

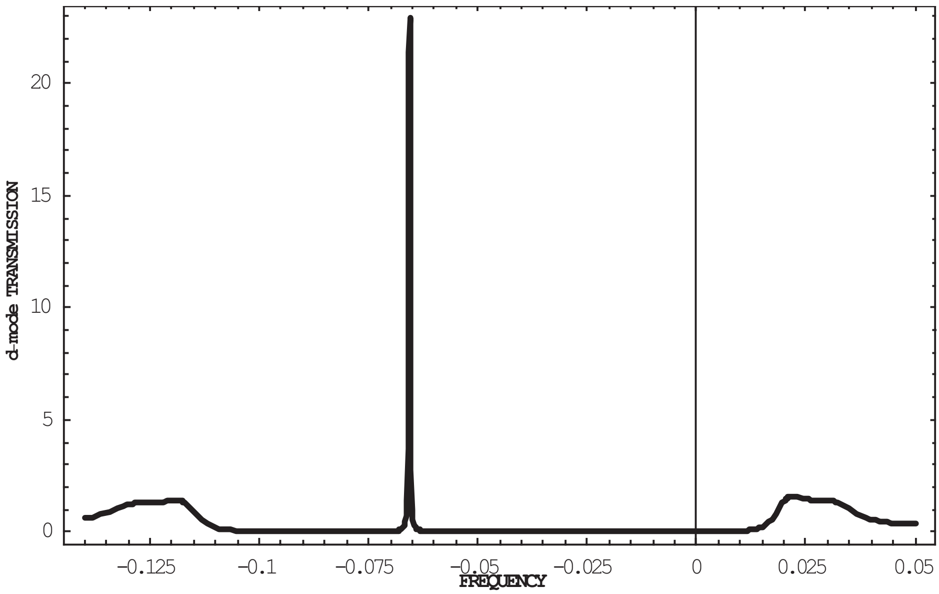


(b)

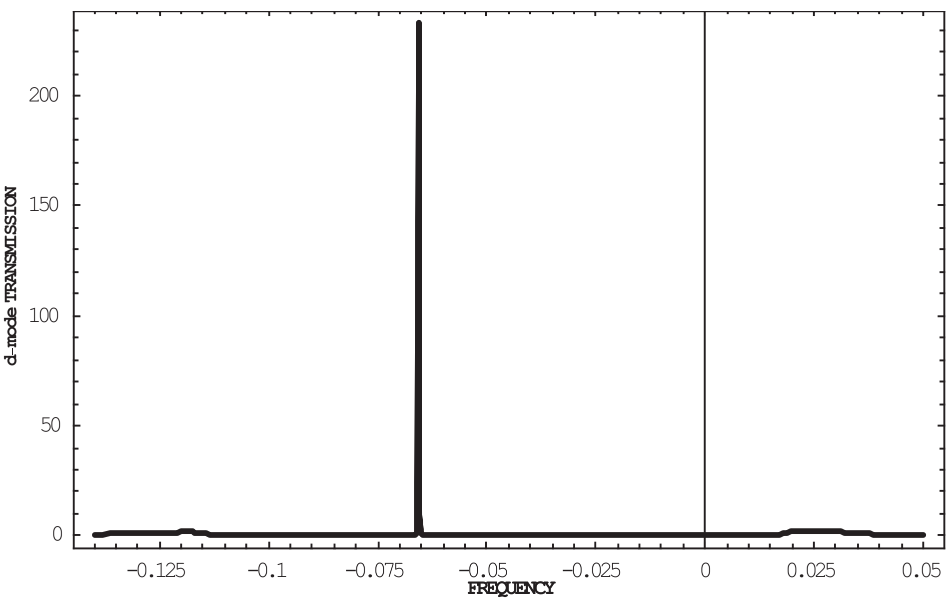
Figure 4. Calculated intensity transmission coefficients at a low birefringent defect layer for an amplifying CLC layers versus the frequency close to their divergence points as a function of γ for diffracting incident polarization at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/20$, $\gamma = -0.00075$ (a), $\pi/16$, $\gamma = -0.00085$ (b), $\pi/12$, $\gamma = -0.00100$ (c), $\pi/8$, $\gamma = -0.00150$ (d), $\pi/6$, $\gamma = -0.002355$ (e), $\pi/4$, $\gamma = -0.003555$ (f), $\pi/2$, $\gamma = -0.004500$ (g), and $\Delta\varphi = 0$, $\gamma = -0.000675$ (h) corresponding to an isotropic defect layer; d/p = 2.25. *(Continued)*

(13). Another option (see [12,13]) is studying of reflection and transmission coefficients (1–2) as a function of γ.

For amplifying CLC the value of γ corresponding to the divergence of DMS reflection and transmission coefficients just determines the solution of the dispersion equation (13)

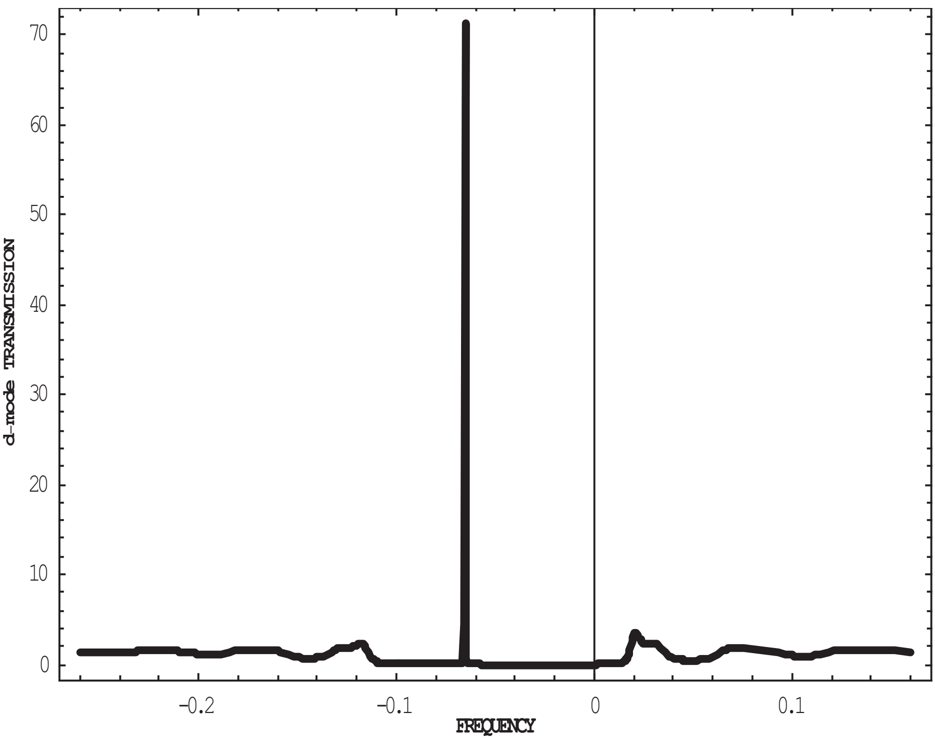


(c)

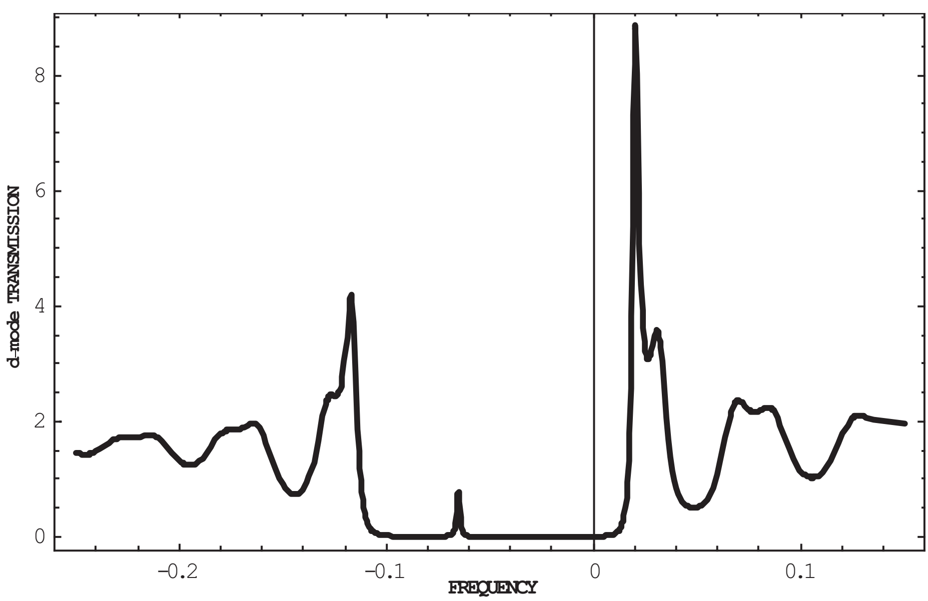


(d)

Figure 4. *(Continued)*



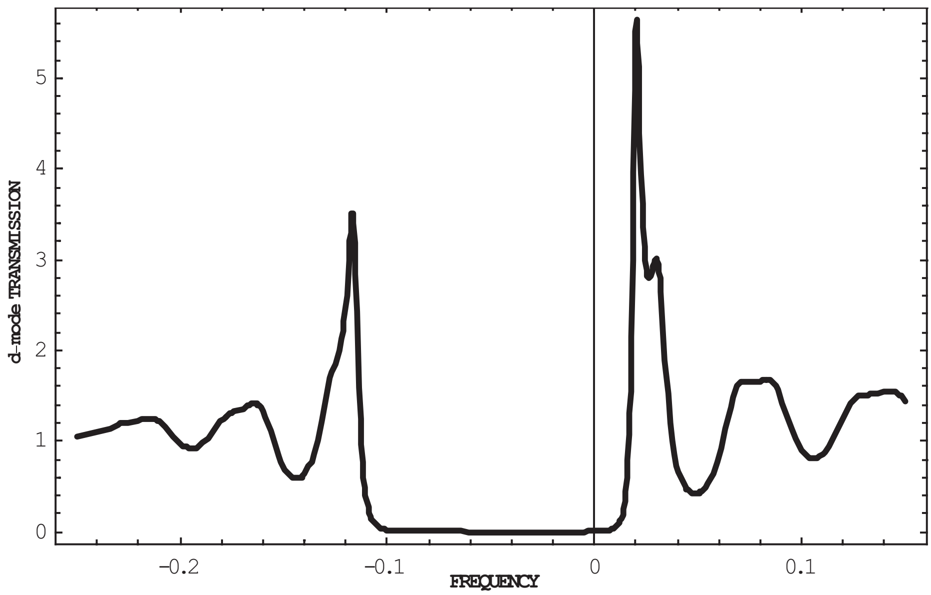
(e)



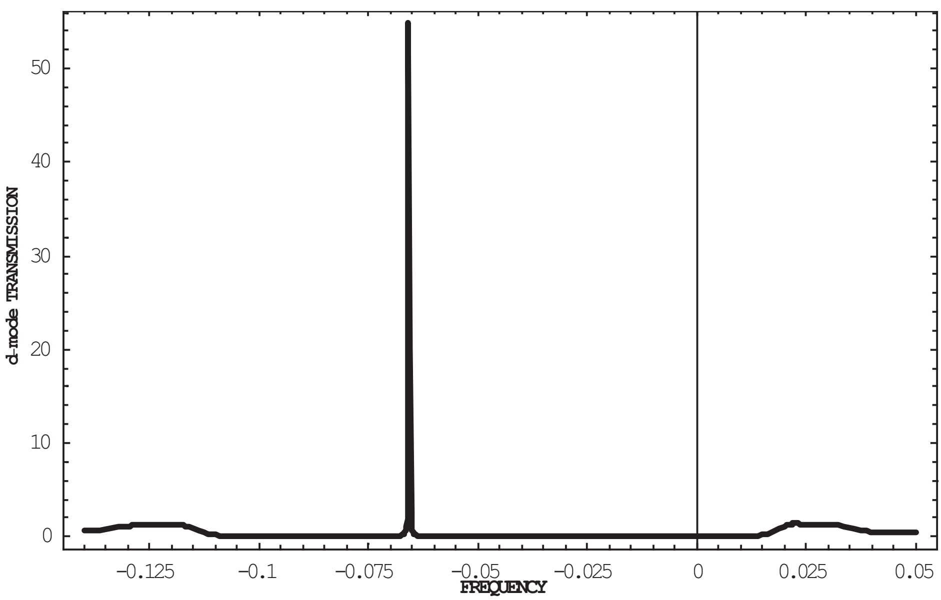
(f)

Figure 4. *(Continued)*

and also determines the threshold DFB lasing gain in the DMS (see [12,13]). So, there is an option to finding the threshold value of γ by calculating the DMS reflection and transmission coefficients at various γ and finding its value at the points of DMS reflection and transmission coefficients divergence.



(g)



(h)

Figure 4. (Continued)

This procedure performed here for a birefringent defect layer at various values of the birefringent phase factor $\Delta e\tau$ related to the light single propagation in a birefringent defect layer allows one to trace the threshold lasing gain (γ) dependence on the birefringent phase factor $\Delta e\tau$. Fig. 4 presenting values of DMS transmission coefficient close to their divergence points demonstrate growth of the threshold DFB lasing gain ($|\gamma|$) with increase of the birefringent phase factor $\Delta e\tau$ and even disappearance of the divergence at defect mode frequency at $\Delta e\tau = \pi/2$. This is in a good agreement with calculated at Figs. 2 and 3, transmission spectra. In particular, at $\Delta e\tau = \pi/2$ there are no traces of the typical for DM peculiarities in transmission spectra.

For absorbing CLC layers in DMS the abnormally strong absorption effect reveals itself at the value of γ ensuring a maximum of the total absorption in the DMS (see [12,13]).

For finite thicknesses of CLC layers L ω_D occurs to be a complex quantity which may be found by a numerical solution of Eq. (13). In particular, positions of dips in reflection and spikes in transmission inside the stop-band just correspond to Re[ω_D] and this observation is very useful for numerical solution of the dispersion equation. What is concerned of the DM life-time it reduces for absorbing CLC layers compared to the case of nonabsorbing CLC layers [12,13].

Conclusion

The performed in the previous sections analytical description of the defect modes at a birefringent defect layer allows one to reveal clear physical picture of these modes which is applicable to the defect modes in general and to reveal options for varying the DM characteristics by controlling the defect layer birefringence. For example, revealed for a birefringent defect layer more low lasing threshold and more strong absorption (under the conditions of anomalously strong absorption effect) for the defect mode frequency at the middle of stop-band compared to the defect mode frequency close to the stop-band edge are the features of any periodic media. For a special choice of the parameters in the experiment the obtained formulas may be directly applied to the experiment. In particular, for the case when the phase difference for the waves of eigen polarizations in their propagation at the defect layer thicknesses d is an integer number of 2π the DM properties are identical to those for a DM at isotropic defect layer. However, in the general case one has take into account a mutual transformation of the two circular polarizations of opposite sense in DMS varying the DM properties. This polarization conversion phenomenon adds also a contribution to the frequency width of the defect mode.

In the conclusion should be stated that the results obtained here for the defect modes with a birefringent defect layer not only clarify the physics of these modes but also open new options for varying the DM characteristics. An important result relating to the DFB lasing at DMS with birefringent defect layer may be formulated as the following. The lasing threshold gain increases with the increase of the optical path difference at the defect layer thickness. The similar result relates to the effect of anomalously strong absorption phenomenon where the value of maximal absorption is dependent on the optical path difference at the defect layer thickness. Note that the obtained above results for the DMS including CLC layers are qualitatively applicable to the corresponding localized electromagnetic modes in any periodic media and may be regarded as a useful guide in the studies of the localized modes with a birefringent defect layer in general.

Acknowledgments

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